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Technical Notes

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Optimized Upwind Dispersion-Relation-Preserving Finite Difference Scheme for Computational Aeroacoustics

M. Zhuang* and R. F. Chen†
Michigan State University, East Lansing, Michigan 48824

I. Introduction

MANY compact and noncompact optimized schemes were recently reviewed by Zingg.[1] Most optimized schemes, however, are restricted to central difference algorithms. This restriction inevitably leads to stability problems that must be dealt with through the use of filters or explicit dissipation terms. Although deliberate filters or dissipation terms have proved quite successful in many acoustic problems,[2,3] in the present study the optimized upwind dispersion-relation-preserving (DRP) finite difference scheme is developed to improve the quality of the numerical solution of short waves without adding an explicit artificial damping term to the finite difference equations.

II. Optimized Upwind DRP Scheme

The optimized upwind DRP scheme developed here is based on the idea of the DRP scheme proposed by Tam and Webb.[4] Consider the approximation of the first spatial derivative $\partial u/\partial x$ by the finite difference for a uniform grid of spacing Δx. Suppose M values of u to the right and N values of u to the left ($N \neq M$) of the point x are used in the finite difference, where x is a continuous variable, i.e.,

$$\frac{\partial u}{\partial x}(x) \simeq \frac{1}{\Delta x} \sum_{j=-N}^{M} a_j u(x + j\Delta x) \tag{1}$$

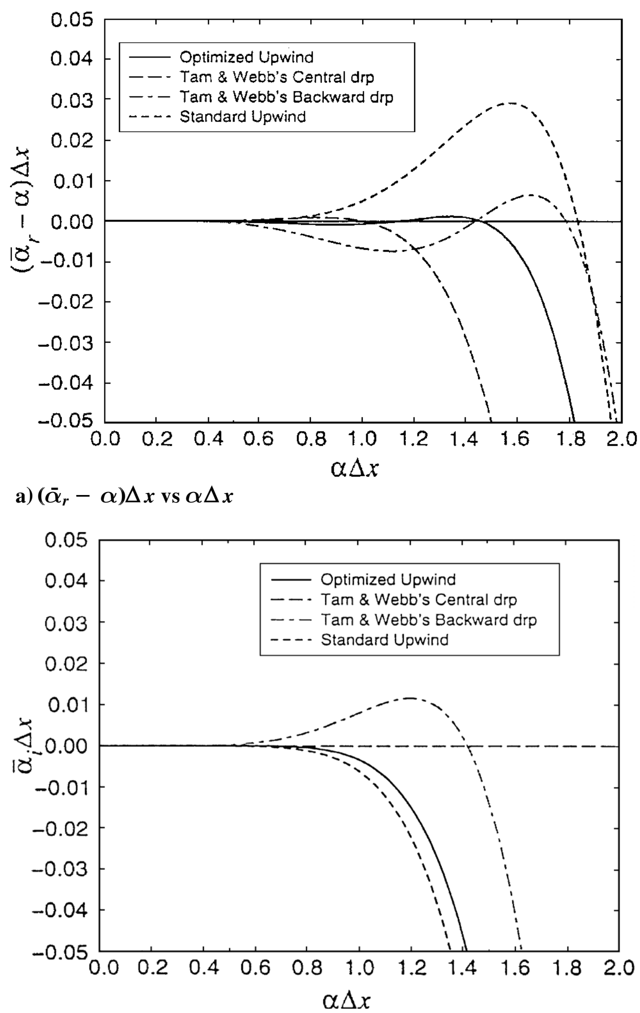
After the Fourier transform of both sides of the preceding equation, the effective numerical wave number of the finite difference scheme can be calculated by[4]

$$\bar{\alpha} = -\frac{i}{\Delta x} \sum_{j=-N}^{M} a_j\, e^{ij\alpha\Delta x} \tag{2}$$

To ensure that the Fourier transform of the finite difference scheme is a good approximation of that of the partial derivative over the range of wave numbers of interest, it is required that a_j in Eq. (2) be chosen so that the effective numerical wave number $\bar{\alpha}$ is close to the wave number α for a wide range of wave numbers. The coefficient of a_j is determined by imposing the condition that Eq. (2) be accurate to the order of $\Delta x^{(M+N-2)}$ through Taylor expansion. This leaves two of the coefficients, e.g., a_{-N} and a_{-N+1}, as free parameters. These parameters are then chosen to minimize the integrated error E ($\partial E/\partial a_j = 0$), defined as

$$E = \int_0^{\pi/2} |\bar{\alpha}_r \Delta x - \alpha \Delta x|^2 \,\mathrm{d}(\alpha\Delta x) + \lambda \int_0^{\pi/2} \left| \bar{\alpha}_i \Delta x + \mathrm{sgn}(c) \exp\left[-\ln 2 \left(\frac{\alpha\Delta x - \pi}{\sigma} \right)^2 \right] \right|^2 \mathrm{d}(\alpha\Delta x) \tag{3}$$

where λ and σ are the adjustable positive constants, $\bar{\alpha}_r$ and $\bar{\alpha}_i$ are the real and imaginary part of $\bar{\alpha}$, and c is the speed of wave propagation in $u_t + cu_x = 0$, the first-order linear wave equation. The optimization process has to allow the values of $\bar{\alpha}_r \Delta x - \alpha\Delta x$ and $\bar{\alpha}_i \Delta x$ to be as close to zero as possible for a wide range of wave numbers. The term $\bar{\alpha}_i \Delta x$ approaches zero in a way specified in the second term of Eq. (3). A similar upwind scheme was studied by Li[5] with a different approach for optimization. Unlike the central



a) $(\bar{\alpha}_r - \alpha)\Delta x$ vs $\alpha\Delta x$

b) $\bar{\alpha}_i \Delta x$ vs $\alpha\Delta x$

Fig. 1 Numerical dispersion and dissipation errors.

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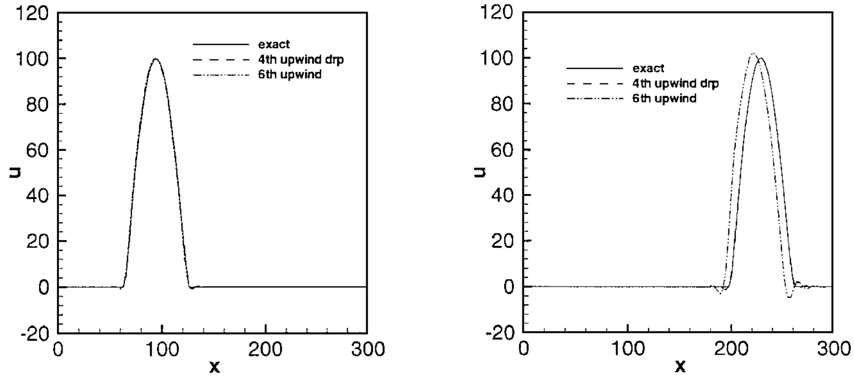
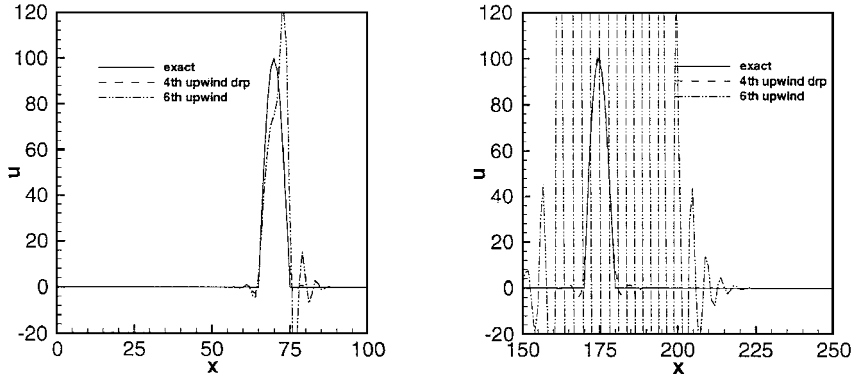
*Assistant Professor, Department of Mechanical Engineering. Member AIAA.

†Graduate Assistant, Department of Mechanical Engineering; currently Project Engineer, Research Division, CFD Research Corporation, Huntsville, AL 35805.



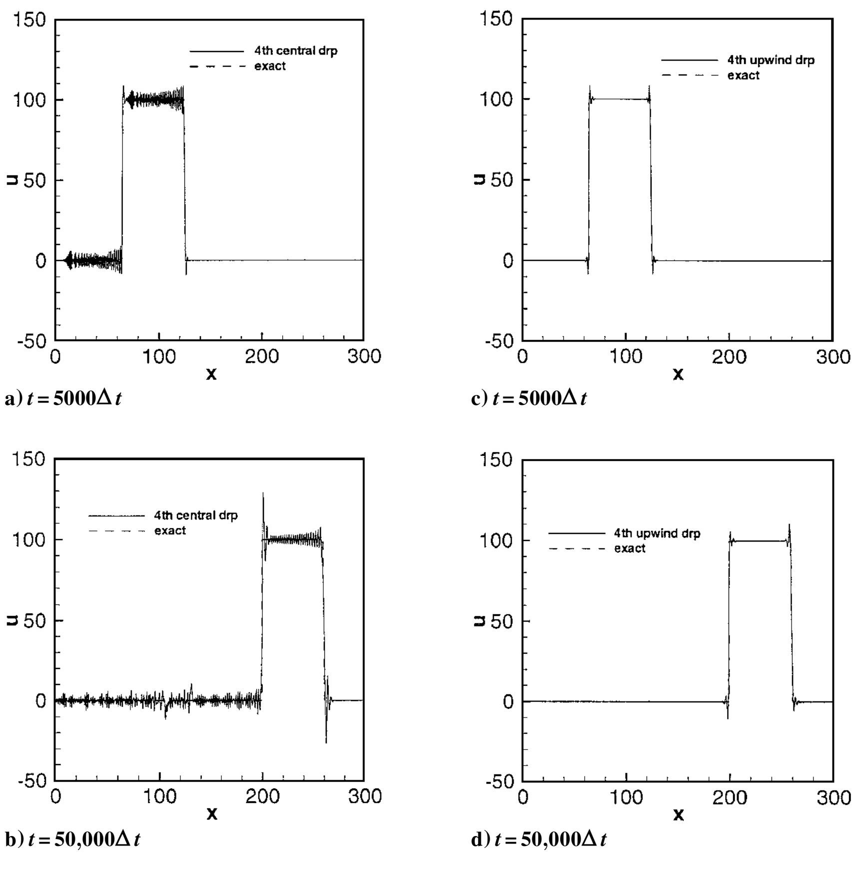
a) $t = 500\Delta t$ b) $t = 5000\Delta t$

Fig. 2 Solution of the first-order linear wave equation for case 1 with $b = 60$ and $\Delta t = 0.0001$.



a) $t = 500\Delta t$ b) $t = 4000\Delta t$

Fig. 3 Solution of the first-order linear wave equation for case 1 with $b = 10$ and $\Delta t = 0.0001$.



a) $t = 5000\Delta t$ c) $t = 5000\Delta t$

b) $t = 50{,}000\Delta t$ d) $t = 50{,}000\Delta t$

Fig. 4 Solution of the first-order linear wave equation for case 2 with $\Delta t = 0.00001$: comparisons of the results of the optimized upwind DRP and the central DRP schemes to those of the exact solutions.

DRP scheme, the advantage of the optimized upwind DRP scheme is that it automatically damps out the spurious short waves while retaining the DRP property in the scheme. The dissipation error is also minimal due to the optimization process. The results in Figs. 1a and 1b indicate that the optimized fourth-order upwind DRP scheme with $N=4$ and $M=2$ has much less dissipation error than that of the standard sixth-order upwind scheme with $N=4$ and $M=2$ and can also resolve waves with higher wave numbers (short waves). The coefficients a_j for the preceding optimized upwind DRP scheme with $N=4$ and $M=2$, shown in Fig. 1, were obtained with $\lambda=0.0374$ and $\sigma=0.2675\pi$. They are $a_{-4}=1.61404967150957E{-}02$, $a_{-3}=-1.22821279019864E{-}01$, $a_{-2}=4.55332277706221E{-}01$, $a_{-1}=-1.24925958826149$, $a_0=5.01890438019346E{-}01$, $a_1=4.39932192729636E{-}01$, and $a_2=-4.12145378889463E{-}02$.

III. Results and Discussion

To verify the behavior of the optimized upwind DRP scheme developed in the preceding section, the scheme, combined with a fourth-order explicit time discretization, is applied to solve the first-order linear wave equation.

In case 1, the half-period sine function is

$$u(x,0)=0, \qquad 0\le x\le 50$$

$$u(x,0)=100(\sin\{\pi[(x-50)/b]\}), \qquad 50\le x\le 50+b$$

$$u(x,0)=0, \qquad 50+b\le x\le 300$$

where b is proportional to the width of the wave.

In case 2, the discontinuous initial waveform is

$$u(x,0)=0, \qquad 0\le x\le 50$$

$$u(x,0)=100.0, \qquad 50\le x\le 110$$

$$u(x,0)=0, \qquad 110\le x\le 300$$

A. Comparisons with the Standard Sixth-Order Upwind Scheme

The first-order linear wave equation is solved for case 1 with $b=60$. Figures 2a and 2b give a comparison between the exact solutions and the numerical solutions for the optimized fourth-order upwind DRP scheme and the standard sixth-order upwind scheme, respectively. The computational domain is given as $0\le x\le 300$, and a mesh of spacing $\Delta x=3$ is chosen. Excellent agreement between the computed solutions of the optimized upwind DRP scheme and the exact solutions is shown for different times, $500\Delta t$ and $4000\Delta t$, where Δt is 0.0001. For the standard sixth-order scheme, however, the phase error increases as the time progresses. As we decrease the value of b, which is known to be proportional to the width of the wave, to 10, mesh spacing Δx has to be decreased to unity. The results shown in Figs. 3a and 3b indicate that for short waves the optimized upwind DRP scheme, due to the DRP property in the scheme, gives good velocity waveform. However, the dispersive error increases for the standard sixth-order upwind scheme, and the solution of the scheme is totally dispersed at the time $4000\Delta t$.

B. Comparisons with the Central DRP Scheme

To compare the performance of the optimized fourth-order upwind DRP with that of the fourth-order central DRP schemes, the first-order linear wave equation is solved for a discontinuous initial waveform (case 2) with a grid of spacing $\Delta x=0.5$. Figures 4a and 4b show the computed waveform at times $5000\Delta t$ and $50{,}000\Delta t$ by the central DRP scheme, where Δt is 0.00001. As we can see, the solution quality degrades by the presence of numerous fine-scale oscillations. The dispersive waves can also be seen superimposed on the main wave pulse. An artificial selective damping is needed to remove these pollutants of computational acoustics.[2,3] In contrast to the central DRP scheme, the results shown in Figs. 4c and 4d indicate that the solution of the optimized upwind DRP scheme has fewer oscillations and most of the spurious waves are automatically damped out due to the inherent viscosity in the scheme. Also, the dissipation error of the optimized upwind DRP scheme is minimum because the imaginary part of the integrated error E is minimized for a wide range of wave numbers.

IV. Conclusions

The optimized upwind DRP scheme is developed and validated. Results from the optimized fourth-order upwind DRP scheme show that numerical solutions of the scheme have much smaller dispersive and dissipation errors and can resolve waves with much shorter wavelengths than that of the standard sixth-order upwind scheme. Compared with the central DRP scheme, the advantage of the optimized upwind DRP scheme is that it removes most of the numerical contaminants of computational acoustics automatically due to the inherent viscosity in the scheme. The quality of numerical solutions is improved by the optimized upwind DRP scheme without adding artificial selective damping terms in finite difference equations.

Acknowledgment

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D. S. McRae
Associate Editor

Finite Element and Dynamic Stiffness Methods Compared for Modal Analysis of Composite Wings

Mark Lillico* and Richard Butler†
University of Bath, Bath BA2 7AY, England, United Kingdom

Introduction

THE aim of the research, which is partly described in this Note, is to develop a design tool[1,2] for use by engineers involved in the conceptual design of wing structures. During the conceptual stage, there is only a limited amount of design information available, making the generation of complex three-dimensional finite element (FE) models difficult. There is also a need to examine numerous design configurations quickly. Therefore, it is important to have a tool that is accurate, efficient, quick to use, and simple, i.e., has a small number of design variables and a sufficient number of critical constraints. Also, because changes made late in the design process are expensive to carry out, it is important to include the effect of constraints that often are neglected at the conceptual stage, e.g., flutter. For these reasons, a one-dimensional dynamic stiffness method (DSM)[3] model has been used instead of a three-dimensional finite element method (FEM) model to find the normal modes of the wing. In previous work, it has been shown, by both experimental and FEM analysis, that the DSM can be used to accurately predict the normal modes

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*Research Assistant, Department of Mechanical Engineering.

†Lecturer in Aircraft Structures, Department of Mechanical Engineering.